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A Modified Preisach Model for Hysteresis in Piezoelectric Actuators

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***Abstract*—Piezoelectric actuators (PEAs) exhibit hysteresis nonlinearity in open-loop operation, which may lead to undesirable inaccuracy and limit system performance. Classical Preisach model is widely used for portraying hysteresis but it requires a large number of first-order reversal curves to ensure the model accuracy. All the curves may not be obtained due to limitations of experimental conditions, and the detachment between the major and minor loops is not taken into account. This paper aims to propose a modified Preisach model that demands relatively few measurements and that describes the detachment. The modified model is implemented by adding a damper in parallel with the classical Preisach model. The parameter of the damper is adjusted to an appropriate value so that the measured and predicted hysteresis loops are in good agreement. Experimental results prove that the proposed modified Preisach model can characterize hysteresis more accurately than the classical model.**

Keywords—hysteresis; Preisach model; piezoelectric actuator; damper

I. INTRODUCTION

Piezoelectric actuators (PEAs) are commonly used in a variety of applications [1-4]. However, PEAs also exhibit adversary hysteresis effect in open-loop operation. Hysteresis, which is observed between the voltages applied to the actuator and the obtained displacement as seen as in Fig. 1, is a nonlinearity that the obtained displacement depends on the present input voltage as well as on how the inputs were applied previously. Hysteresis severely limits system performance, since it leads to a positioning uncertainty of up to 15% of motion range [5].

To compensate hysteresis by open-loop control, a model precisely modeling hysteresis is needed. Several models have been developed and reported. Amongst those models Preisach [6, 7] and Prandtl-Ishlinskii (PI) [8] models are formed by a weighed superposition of infinite elementary hysteresis operators. Duhem model [9] is in the form of nonlinear differential equation. These models do not describe the physical mechanism of hysteresis, thus then are referred to phenomenological models. In comparison, the model proposed in [10] is constructed by physical consideration in which the basic physical elements including springs, massless carts, and Coulomb frictions are employed to portray hysteresis.

Classical Preisach model is widely used for portraying hysteresis, but the model accuracy is dependent on the number of measured first-order reversal curves of hysteresis loop and, due to the limitations of experimental conditions, it is possible that the number of obtained curves is not enough to accurately model hysteresis. Moreover, the detachment between the major and minor loops is not taken into account in the classical Preisach model. To overcome these limitations, this paper proposes a modified Preisach model to accurately characterize hysteresis in PEAs. This model requires relatively few measured first-order reversal curves and describes the attachment. In the proposed model, a damper is added in parallel with the classical Preisach model. The parameter of the damper is adjusted to an appropriate value so that the measured and predicted hysteresis loops are in good agreement. Experimental results show a significant reduction of the relative error between the measured and predicted hysteresis response curves by using the modified Preisach model.

II. CLASSICAL PREISACH MODEL

A. Model description and implementation

The basic idea of the Preisach model lies in the description of the hysteresis through an infinite number of operators $\gamma_{\alpha\beta}$. Consider a simple two-position relay operator $\gamma_{\alpha\beta}$, in which the input-output relationship at each instant of time t as depicted in Fig. 2 can be described by:

$$\gamma_{\alpha\beta}(u(t)) = \begin{cases} 0 & if \quad u(t) \le \beta \\ 1 & if \quad u(t) \ge \alpha \\ u(t) & if \quad \beta < u(t) < \alpha \end{cases} \tag{1}$$

where numbers α and β correspond to "up" and "down" switching values of input, respectively (assumed in the sequel that $\alpha \le \beta$).

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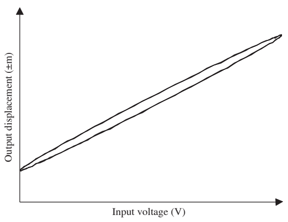


Fig. 1. Hysteresis in piezoelectric actuators

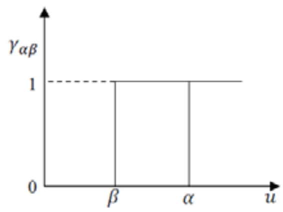


Fig. 2. An elementary two-position relay operator

Along with the relay operator $\gamma_{\alpha\beta}$, consider a weighting function $\mu(\alpha,\beta)$. Then the mathematical form of the Preisach model can be written as follows [11]:

$$x(t) = \iint_{\alpha \geq \beta} \mu(\alpha,\beta)\gamma_{\alpha\beta}(u(t))d\alpha d\beta \qquad (2)$$

where $x(t)$ is the output displacement of a PEA.

The mechanism of memory formation in the Preisach model is usually discussed in the Preisach (α,β) plane, $P \triangleq \{(\alpha,\beta) \in P \mid \alpha \geq \beta, \alpha \leq \alpha_0, \beta \geq 0\}$, as shown in Fig. 3(a). Each $(\alpha,\beta) \in P$ corresponds to an operator $\gamma_{\alpha\beta}$. It is assumed that the weighting function $\mu(\alpha,\beta)$ is equal to zero outside of P. At each time instant, P is divided into two parts:

$$P_- \triangleq \{(\alpha,\beta) \in P \mid \text{output of } \gamma_{\alpha\beta} \text{ at t is } 0\},$$

$$P_+ \triangleq \{(\alpha,\beta) \in P \mid \text{output of } \gamma_{\alpha\beta} \text{ at t is } 1\},$$

so that $P = P_-(t) \cup P_+(t)$ at any time. Equation (2) can be replaced by:

$$x(t) = \iint_{P_+(t)} \mu(\alpha,\beta)d\alpha d\beta \qquad (3)$$

Assume that at an initial time t_0, the input $u(t_0) = u_0 < 0$. This means that the output of every relay operator $\gamma_{\alpha\beta}$ is 0, and $P_-(t_0) = P$, $P_+(t_0) = 0$ (Fig. 3(b)). Next as the input is monotonically increasing to some maximum value at t_1 with $u(t_1) = u_1$, all the relay operators $\gamma_{\alpha\beta}$ with switching values α less than the current input u_1 are being switched to 1. Thus at time t_1 the boundary between $P_-(t)$ and $P_+(t)$ is the horizontal line $\alpha = u_1$ (Fig. 3(c)). Now, as the input starts to decrease monotonically until it stops at t_2 with $u(t_2) = u_2$, all the relay operators $\gamma_{\alpha\beta}$ with switching values β larger than the current input u_2 become 0 and a vertical line segment $\beta = u_2$ is generated as part of boundary (Fig. 3(d)). Then additional horizontal or vertical boundary segments are generated as input reversals.

From above analysis, it can be seen that the output of the Preisach plane is determined by the boundary link between P_- and P_+. Due to its staircase structure, the boundary link is fully captured by all corner points, which correspond exactly to the past dominant maximum and minimum input values and that are stored and updated in numerical implementation of the Preisach model.

A displacement function $\delta(\alpha',\beta')$ is introduced in the identification of Preisach model, as follows:

$$\delta(\alpha',\beta') = x(\alpha') - x(\alpha',\beta') \qquad (4)$$

where - $\delta(\alpha',\beta')$ represents the change in the output $x(t)$ as the input $u(t)$ changes from α' to β'; - $x(\alpha')$ is the output value on the limiting ascending branch of the first-order reversal curve and - $x(\alpha',\beta')$ is the output value on the descending curve, which is attached to the limiting ascending branch at the point $x(\alpha')$ (Fig. 4). α' and β' represent the maxima and the minima of the input $u(t)$.

A geometrical interpretation for $x(\alpha')$ and $x(\alpha',\beta')$ can be illustrated in the Preisach plane shown in Fig. 5. It can be seen that the result of the input $u(t)$ decreasing from α' to β' leads to subtracting a triangle $T(\alpha',\beta')$ from the region P_+. Thus Equation (4) can be written as:

$$\delta(\alpha',\beta') = \iint_{T(\alpha',\beta')} \mu(\alpha,\beta)d\alpha d\beta \qquad (5)$$

If the hysteresis loop contains several extrema, the region P_+ of the Preisach plane is composed of several trapezoidal parts P_k, as shown in Fig. 6. All the extrema α_k' and β_k' that depend on the past values of the input voltage $u(t)$ are stored in the history. For the part P_1, the following equation is deduced:

$$\iint_{P_1} \mu(\alpha,\beta)d\alpha d\beta = \delta(\alpha_1',\beta_0') - \delta(\alpha_1',\beta_1') \qquad (6)$$

The other parts are calculated in the same way. Because the integration on the region P_+ is the sum of the integrations on all the parts P_k, the total displacement of the PEA $x(t)$ for an input voltage $u(t)$ is determined thanks to (3), depending on the current slope of $u(t)$:

$\dot{u}(t) > 0$

$$x(t) = \sum_{k=1}^{N} [\delta(\alpha_k',\beta_{k-1}') - \delta(\alpha_k',\beta_k')] + \delta(u(t),\beta_N') \qquad (7)$$

$\dot{u}(t) < 0$

$$x(t) = \sum_{k=1}^{N-1} [\delta(\alpha_k',\beta_{k-1}') - \delta(\alpha_k',\beta_k')] + \delta(\alpha_N',\beta_{N-1}') - \delta(\alpha_N',u(t)) \qquad (8)$$

where N is the number of maxima α_k' and minima β_k' that are stored.

In order to compute the values $\delta(\alpha',\beta')$, a mesh of α and β is created within P. The reference value $\delta(\alpha,\beta)$, are measured on the PEA for all α and β of the mesh and stored at each corresponding node as depicted in Fig. 7. Once the cell in which a given pair (α',β') lies is determined, the corresponding value $\delta(\alpha_i',\beta_j')$ is computed using a bilinear spline interpolation:

$$\delta(\alpha',\beta') = a_0 + a_1\alpha' + a_2\beta' + a_3\alpha'\beta' \qquad (9)$$

data pairs to collect and store, which requires more measured first-order reversal curves.

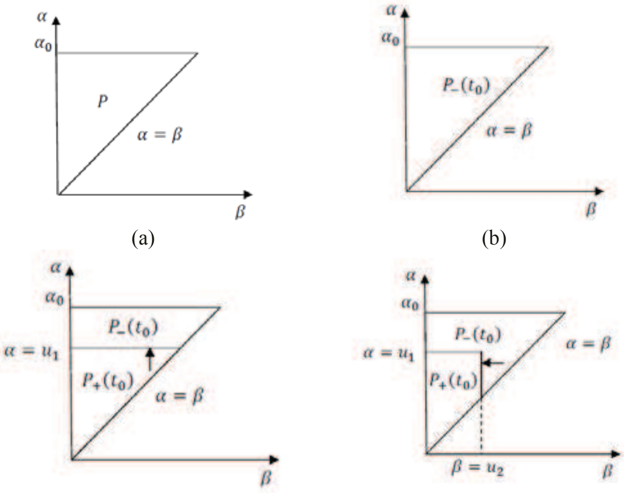


Fig. 3. Memory curve in the Preisach plane

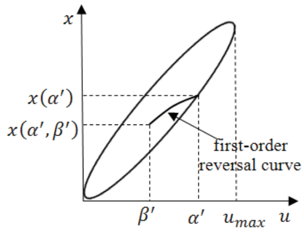


Fig. 4. First-order reversal curve

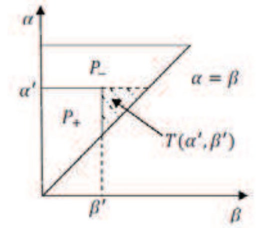


Fig. 5. Illustration of $T(\alpha^{'},\beta^{'})$

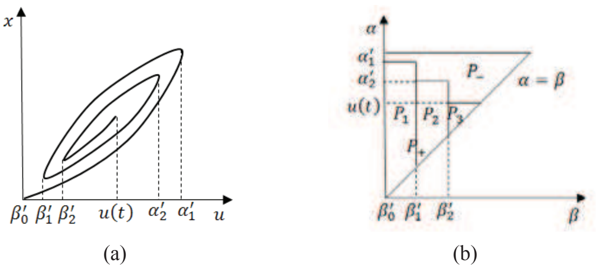


Fig. 6. (a) Hysteresis loop with several extrema $\alpha_k^{'}$ and $\beta_k^{'}$ (b) the region P_+ corresponding to (a)

For every $\delta(\alpha^{'},\beta^{'})$, the interpolation coefficients a_0, a_1, a_2 and a_3 are obtained through the same spline interpolation based on the values of the nodes surrounding the cell $\delta(\alpha_i^{'},\beta_j^{'})$, $\delta(\alpha_i^{'},\beta_{j+1}^{'})$, $\delta(\alpha_{i+1}^{'},\beta_j^{'})$ and $\delta(\alpha_{i+1}^{'},\beta_{j+1}^{'})$ as shown in Fig. 7. The output displacement is determined using either (7) or (8).

The wipe-out property is needed in the Preisach model. It allows erasing the pair $(\alpha_N^{'},\beta_{N-1}^{'})$ from the history once $u(t)$ exceeds $\alpha_N^{'}$. Similarly, the pair $(\alpha_N^{'},\beta_N^{'})$ can be erased from the history once $u(t)$ becomes smaller than $\beta_N^{'}$. This avoids the excessive growing of the stored values.

From above procedure of identification, it is obviously seen that the accuracy of the model depends on the mesh density of the Preisach plane. The denser mesh means more data pairs to collect and store, which requires more measured first-order reversal curves.

B. Experiments

The model implementation and validation need many experiments that are realized in open-loop. The main components of experimental setup are shown in Fig. 8. A ring-type piezoelectric actuator (NCE41, Noliac) with 400V maximum operating voltages is used as study object. The displacement sensor is a noncontact capacitive sensor (D510.100, PI) enabling the capture of the PEA performances. Fig. 9 shows a functional diagram of experimental system.

To implement the classical Preisach model, a set of piecewise sinusoidal voltages of amplitude range from 10 V to 400 V with 10 V increment (Fig. 10) is applied to the PEA. A frequency of 0.25 Hz is chosen in the experiment so that neither creep nor dynamic effect of piezoelectric actuator affects the output response. In total, 861($41\times(41+1)/2$) data pairs ($\delta(\alpha^{'},\beta^{'})$) measured from the experiment are collected and stored in the computer. Then the classical Preisach model is implemented.

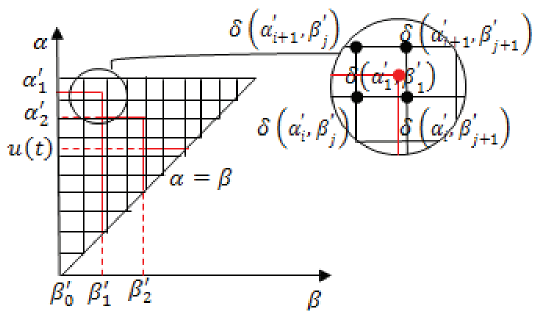


Fig. 7. Discretization of the Preisach plane P into a finite number of rectangles

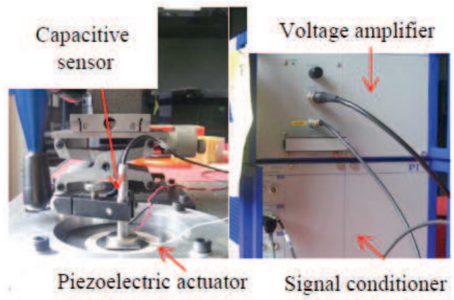


Fig. 8. The experimental setup

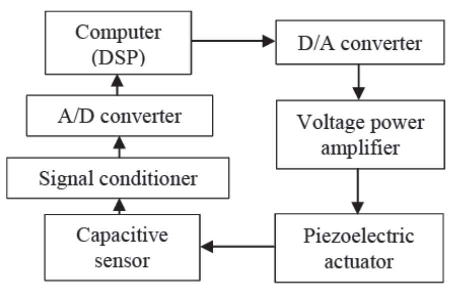


Fig. 9. Diagram of experimental system

To evaluate the model accuracy, two sets of sinusoid input voltages respectively with amplitude 200 V and 100 V are applied to the PEA. Fig. 11(a)-(b) and Fig. 12(a)-(b) illustrate the measured and predicted hysteresis loops and time responses. Figures show that the predicted curves computed by using classical Preisach model do not conform very well to the measured curves in the PEA and that, for both cases, the predicted hysteresis loops are thinner than the measured loops. Fig. 11(c) and Fig. 12(c) depict the relative errors between the measured and predicted responses. It is observed that the maximum relative errors for the two cases respectively approach to 4% and 3.5%.

From Fig. 11(a) and Fig. 12(a), it can also be seen that the predicted hysteresis loop with sinusoid input voltages of amplitude 200 V is closer to the measured loop than the case with sinusoid input voltages of amplitude 100 V. This is because more data pairs are employed for the model computation.

In addition to the influence of the number of measured first-order reversal curves on the model accuracy, another factor is analyzed. Fig. 13 shows major and minor hysteresis loops respectively generated by exciting the PEA with positive sinusoid input voltages of 0.25 Hz frequency and of 200 V and 100 V amplitudes. Fig. 13(a) depicts that the measured hysteresis curve of the minor loop detaches inward from that of the major loop. Hence the corresponding data pairs of these inward detachments are collected and stored for the implementation of classical Preisah model. However, the minor loop attaches to the major loop on the basis of classical Preisach model as depicted in Fig. 13(b). That is why the predicted hysteresis loops look like thinner than the measured loops. Concerning the detachment, it is observed that, while the time rate of the input for the minor loop is zero, the rate for the major loop is nearly at its peak. For this reason, it is postulated that the detachment is caused by rate-related dynamics [12]. That is why we propose to add a damper in parallel with the classical Preisach model to describe the detachment and then to get a more accurate model of the hysteresis.

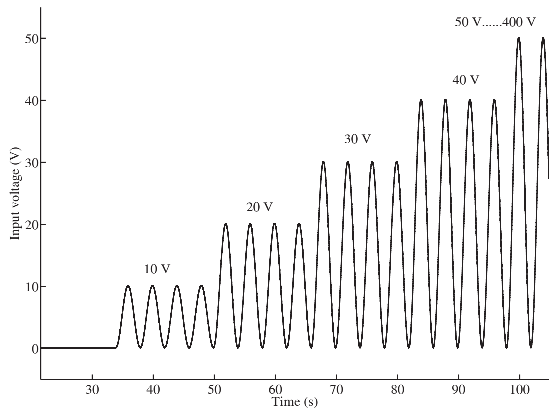
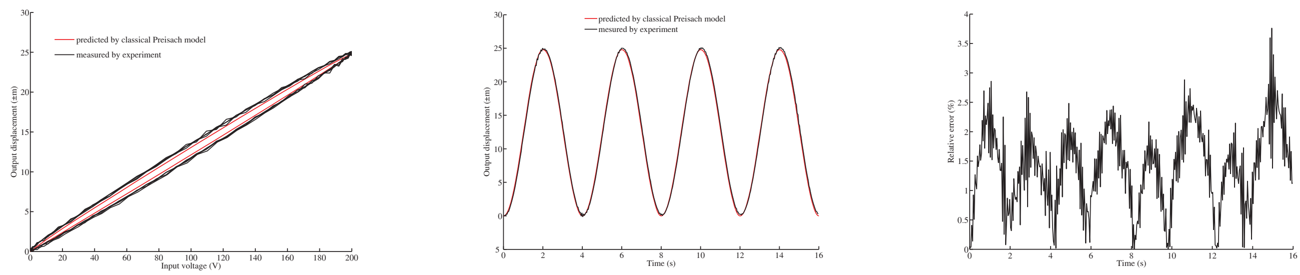
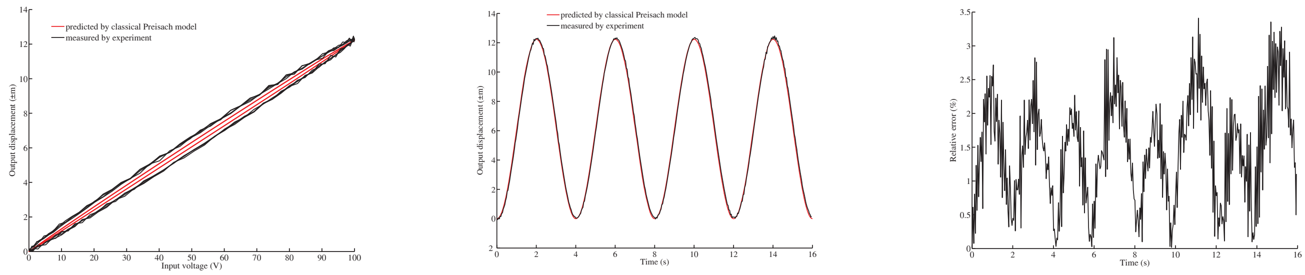


Fig. 10. A sine piecewise voltage signal used to identify classical Preisach model



(a) (b) (c)

Fig. 11. Hysteresis responses with sinusoid voltages of amplitude 200 V (a) hysteresis loops (b) time responses (c) relative error between the measured and predicted curves



(a) (b) (c)

Fig. 12. Hysteresis responses with sinusoid voltages of amplitude 100 V (a) hysteresis loops (b) time responses (c) relative error between the measured and predicted curves

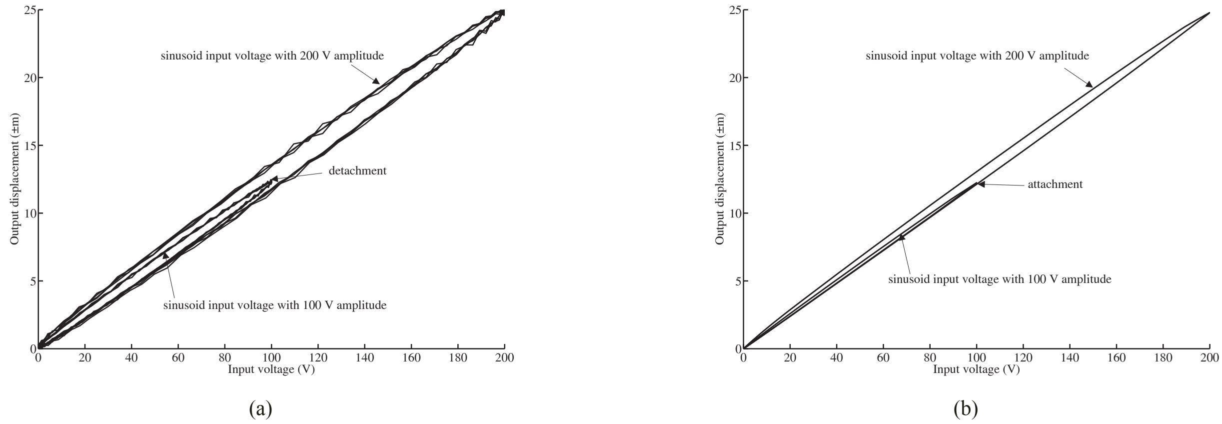


Fig. 13. (a) Measured and (b) predicted (by using classical Preisach model) hysteresis loops with sinusoid voltage of amplitude 200 V and 100 V

III. MODIFIED PREISACH MODEL

In order to model hysteresis more precisely, the classical Preisach model is modified by adding a damper D_m in parallel as shown in Fig. 14. The damper D_m has two effects: 1- to plump the predicted hysteresis loop to decrease the error with the measured loop; 2- to model the detachment phenomenon between the major and minor loops. To identify the numerical value for D_m , the case with sinusoid input voltages of amplitude 200 V is considered as the reference one. D_m is tuned incrementally from zero until the error between the predicted and measured loops is minimal, as shown in Fig. 15(a). Here D_m is chosen to 0.003.

The case with sinusoid input voltages of amplitude 100 V is still studied here and is compared with the reference case of amplitude 200 V. Fig. 15(a)-(b) and Fig. 16(a)-(b) illustrate the measured and predicted hysteresis loops and time responses for both cases. Figures show that the predicted curves computed by using modified Preisach model conform exactly to the measured curves of the PEA for both cases. It means that the value of D_m obtained from the reference case can be used for other input voltage amplitudes. From Fig. 15(c) and Fig. 16(c), it is observed that the maximum relative errors for both cases reach to 2%. Compared to Fig. 11(c) and Fig.12(c), 50% and 42% of the errors vanish respectively. The good agreement found between the measured and predicted curves shows that the modified Presach model can describe the hysteresis in PEAs more precisely and do not require more measurements of the first-order reversal curves.

Finally, Fig. 17 shows the comparison between the measured hysteresis loops and the predicted loops computed by using the modified Preisach model for both studied cases. It is obviously seen that the detachment between the major and minor loops can be precisely described with the modified Preisach model.

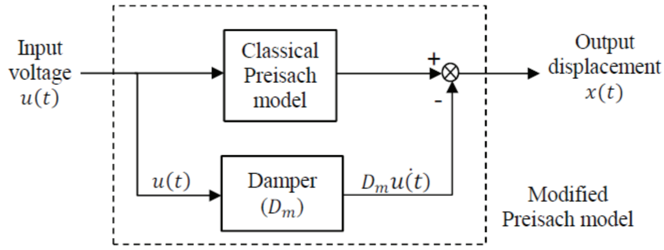


Fig. 14. Modified Preisach model

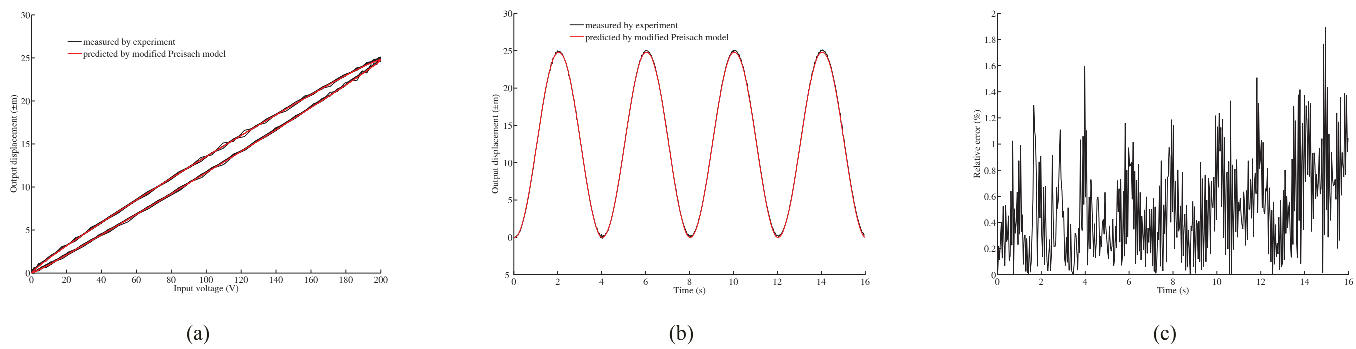
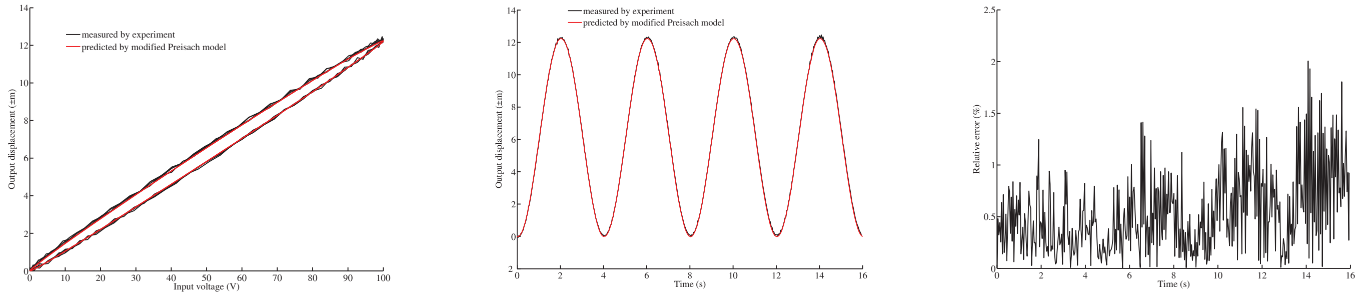


Fig. 15. Hysteresis responses with sinusoid voltages of amplitude 200 V (a) hysteresis loops (b) time responses (c) relative error between the measured and predicted curves



(a) (b) (c)

Fig. 16. Hysteresis responses with sinusoid voltages of amplitude 100 V (a) hysteresis loops (b) time responses (c) relative error between the measured and predicted curves

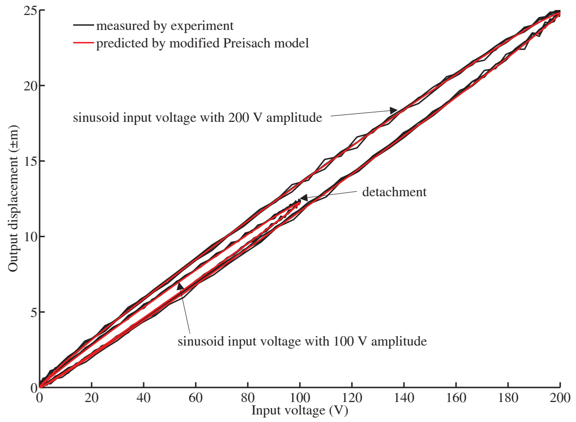


Fig. 17. The detachment in measured and predicted (by using modified Preisach model) hysteresis loops

IV. CONCLUSION

This paper proposes a modified Preisach model that can more accurately portray hysteresis in PEA than classical Preisach model. The proposed model is based on relatively few measured first-order reversal curves and can also characterize the detachment phenomenon between the major and minor loops. The presented modified model is implemented by adding a damper in parallel with the classical Preisach model. The damper is used for plumping the shape of the predicted hysteresis loop to conform to the measured loop and for modeling the detachment that may be caused by rate-related dynamics. Experimental results show that the maximum relative errors between the measured and predicted hysteresis responses is reduced by 50% and by 42% respectively for two studied cases by using the modified model. It is evident that the proposed modified Preisach model is an effective mean to accurately describe hysteresis.

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